Interactive Documents –
Yet Another Way to Activate Students in Mathematics

Thomas Risse

Institute for Informatics and Automation, Hochschule Bremen

Key words: *Tele-learning, active/engaged learning, interactive documents, lecture notes, mathematics, numerics*

Abstract:

It is well known that mathematical reasoning is not very well developed in students starting technical degree courses. Specific problematic learning behaviour is identified preventing engaged and active learning. These phenomena are described and different means to stimulate students are presented.
Interactive documents, i.e. documents that compute, exhibit another way to make students to explore a topic, to try different solutions to a problem, to compare algorithms by cost to benefit analysis.
These interactive documents are pdf documents generated by the free type setting system pdfTeX. Students browse the documents by the free Adobe Acrobat reader. The potential of interactive documents is illustrated by lectures notes of basic numerical mathematics. Further examples and results of an evaluation of the usage of these documents are presented.

1 Introduction

A solid mathematical education in the first year of study is indispensable – this is the credo of the staff in the Department of Electrical and Electronic Engineering and Informatics of Hochschule Bremen. Experiences circumstantiate that students starting technical degree courses often have no sound knowledge of basic mathematics, and their mathematical, i.e. abstract reasoning is not very well developed.

We try to even out these deficits on all levels of mathematical education

1. by offering a preliminary test [21] so that students can check their mathematical abilities and determine whether they need to participate in a preliminary tutorial. In this way, we hope that all students start their mathematical education at our Hochschule more or less on the same level of 'expertise'. This test exploits some of the methods presented here.
2. by experimenting in our lectures with different means of computer support, e.g. learning programs, visualisations, computer algebra systems, virtual experiments, lectures online and on demand etc. Common to all these efforts is to motivate and to activate students. Yet another way to do so in numerical mathematics [20] is presented here.
3. by offering optional courses on computer algebra systems which provide ample opportunities for independent activities of the students.

In mathematics, students tend to nurture a consumers attitude, tend to expect to be entertained, tend to believe that it suffices to watch somebody solving mathematical problems instead of insistently trying to solve it themselves.

Therefore, we have to focus on how to activate students in mathematics, how to improve the teacher-learner interaction, and how to produce the highest sustainable learning outcome.

In this aspect, we present various means to activate the students, most notably interactive lecture notes.

First, let us investigate objectives and problems in more depth and see what interactive documents can contribute to improve the mathematical education at universities of applied sciences as the Hochschule Bremen.

Application orientation implies that the students have to acquire tactical and strategic learning [1], i.e.

- **tactical learning:** students learn rules and procedures to solve problems of types they already known
- **strategic learning:** students learn rules, procedures, approaches, and ways to develop procedures to solve problems of types unknown to them.

Obviously, strategic learning is more demanding than tactical learning. But at the same time, strategic learning represents the skills students should acquire to cope with the broad variety of problems they will encounter in practice.

Of course, computer-algebra-systems like Mathematica [28], Maple, MuPad, MathCad, MatLab, Derive, Reduce, Scratchpad, etc. can support strategic learning. But to our experience students can utilise such systems on a sound mathematical footing and with solid skills only.

1.1 Learning Objectives

Students are to learn to formalise a problem which in practice often is specified only vaguely, to set up models, to state requirements more precisely, to realise how to divide the problem, to limit the space of relevant solutions, and to identify procedures which once applied will lead to solutions or will optimise suboptimal solutions.

1.2 Examples of Applications

Some examples show that the skills to be developed are essential in all areas of engineering.

- Fractal compression or iterative function-systems need affine mappings.
- Coding/encryption like ECC/RSA make use of polynom algebra/number theory.
- Signal and image processing are unimaginable without Fourier transform and its descendants; control is nothing without the Laplace transformation.
- Quality control and assurance avail themselves of probability calculus and statistics, etc.
- Generative computer graphics require physical modelling [2] of the scene to be rendered. Besides physics, geometry, transformation matrices and differential equations together with their numerical algorithms to approximate the solution are in demand.

1.3   Conditions

To our knowledge, teaching and learning conditions for informatics students at Hochschule Bremen are typical for many universities of applied sciences in Germany. The following observations seem to hold in general.

Let us investigate how type of course, financial and personal resources determine the teaching and learning behaviour.

Traditional and Virtual Lectures

Traditionally, education in mathematics takes place in lectures (at best accompanied by tutorials). To support these courses all my colleagues offer materials like lecture notes, black board or whiteboard content, exercises, old exams, link collections, programs etc on web servers, e.g. [10], [12], [17].
It is popular to use learning programs like, for example *Mathe Online* [3], Geonet for geometric constructions [29], *e-Calculus* or *Algebra in 10 Lessons* [22], statistics [7] or a Fourier series unit [9] – to name just a few. Link lists [11], [17] point to more examples.

At Hochschule Bremen, we constantly try out new techniques like *white board systems* [19] or *authoring on the fly* [14] where we produced *online lectures* as well as *lectures on demand*, e.g. a unit on cryptography [18].

Structural Shortcomings

For tutorials there often money or suitable staff is lacking although tutorials represent the indispensable opportunity for guided exercises, for questions, corrections and feedback.

Since decades (see [13], [26] for overview, history and references) researchers especially in the field of artificial intelligence developed *Intelligent tutorial systems, ITS* which provide careful support, taylor made feedback, precise error analysis, hints for independent corrections etc. Unfortunately, there is no established authoring tools and a fortiori no established ITS for mathematics, but at least there are promising new developments [4] in collaboration with our department.

Teaching and Learning Behaviour

Everyday, staff experiences the dilemma of an obligatory syllabus hardly to be mastered – a bit if linear algebra, some analytical geometry, standard calculus, a couple of differential equations, some stochastics – on one hand and of wishes to go on to topics like graph theory, optimisation etc. on the other hand. Staff has to cope with the usually very broad range of students precognition and skills as well as with students expectations of edutainment and infotainment.

For many students, mathematics is a necessary evil. This attitude determines students learning behaviour which varies between two extremes:

- Desirable are curiosity, inquisitiveness, activity, flexibility, creativeness, interdisciplinarity, initiative, ...

- Undesirable are consumers attitude, orientation solely on marks, taking notes mechanically, expecting recipes for well known problems only, tolerating application orientation only if not burdened with too much context like physics, ...

The two extremes can be classified and characterised by fundamental and initial attitude, self-conception, approach as well as by higher level characteristics.

Winners	loosers
motivated, active	passive, unmotivated
curious, inquisitive, initiative	with consumers attitude, expecting to be entertained, motivated, accepting everything
engaged, independent	'good marks' – that is the only problem
active, (self) critic, reflective	unassertive, do not overdo, not monitoring own development, 'math just is not my cup of tea'
networked thinking, looking for links and connecting relations, interdisciplinary	thinking in narrow categories, with blinders, application orientation as additional burden

Unfortunately, cause and effect boost each other by feed back. Motivated students take the initiative and are awarded by success in return strengthening the motivation.
Opposite, students who want to be entertained will avoid engagement with the problem. This attitude deprives them from incentives and forces them to denigrate the subject which in turn cannot offer any motivation.

2 Activating Teaching, Active Learning

Everybody knows the commonplace

<div align="center">

to hear is to forget
to see is to remember
to do is to understand

</div>

Therefore, we value explorative learning and of 'learning by doing', Hence, it is necessary
- to motivate students (secret of true pedagogues)
- to get them interested in the subject (e.g. by attractive application examples)
- to get them active (traditionally by exercises, assignments etc., additionally by demonstration/learning programs, virtual experiments, web links, ...)
- to challenge them (by competition, contests, prices, distinction, ...)
- to bewilder (perturbate) them, i.e. according to the schema theory of the constructivism [5], pp35 to provoke new insights and approaches to let the students find new solutions in unexpected situations.

Of course, these objectives can be met by support of learning programs, computer algebra systems, application specific modelling and visualisation programs and the like.

Besides these mentioned ways and means to support teaching, so called *interactive documents* can contribute to activating teaching and active learning – abiding by the maxim

<div align="center">

*All means to activate the students and to increase interaction
between staff and students only can improve the learning outcome!*

</div>

3   Interactive Lecture Notes

Traditionally, lecture notes are documents (on paper) written to support a certain course and to complement textbooks. Lecture notes may make obsolete to jot down notes during lectures. Recordings of blackboard or whiteboard content obviate the fear to go home barehanded. There are means to convert lecture notes to more or less interactive documents providing

- navigation, table of content, indices, footnotes and internal links – with one mouse click users get the requested information presented.
- links to learning programs, animations or visualisations etc., for example *Mathe online* [3], *e-Calculus* and *Algebra in 10 Lessons* [22], *Mathematica's* [28] *MathWorld* [27] etc.
- exercises or old exams assignments linked to their solutions
- links to link collections [11], [17] or [6]
- (partial) print, copy and paste

Now, by using the right tools the degree of interactivity of lecture notes can be increased significantly.

3.1   Ways to Implement Interactive Documents

The freeware type setting program TeX/LaTeX [23] is well known for the high quality of mathematical texts it generates. There are several TeX distributions for all relevant platforms. Especially lecture notes written with pdfTeX [24] with the hyperref package [16] offer all features listed above. pdfTeX generates pdf-documents to be read, saved, searched by the free Acrobat Reader. Using so called styles like e.g. exerquiz [22], question/answer functionality can be integrated into the documents, e.g. our test [21] for the preliminary course. There are even styles (e.g. web [22] or pdfscreen [15]) to publish the same document both to be read on the screen, to be printed and to be presented using a beamer [25].
By the way, computer algebra systems like Mathematica [28] usually can generate and export TeX sources.

3.2   Interactive Lecture Notes – Documents Which Compute

One can embed (using e.g. pdfTeX [24]) JavaScript-code into pdf documents. Acrobat Reader Version 4.0 or later contains a JavaScript interpreter to process this code.
Classical example for this technique is the scientific pdf calculator [8] written by Hans Hagen demonstrating the potential of this technique.

3.3   Interactive Lecture Notes for Numerics

Of course, all algorithms of numerical mathematics can be implemented in JavaScript. The objective was to make the students to engage in numerics, to experience stability or convergence. As result some little applications were written in order to

- determine the relative precision of computation!
  The algorithm to determine the relative precision is presented. Students then can let the document compute the relative precision of JavaScript applications.

- experience examples for violation of the law of associativity due to limited precision!
  Students check the law of associativity of the JavaScript float addition/subtraction. It is not yet possible for the user to specify any precision.
- experience vector calculus in the plane and in three dimensional space by computation of scalar and vector products etc!
  Students are asked e.g. to check vectors for orthogonality, normality, the included angle, area of triangles, volume of parallelepiped, tetrahedron etc
- solve systems of linear equations per Gauß' Elimination algorithm (with and without pivot) as well as per Gauß-Seidel algorithm!
  Students check solvability and the precision of the solution, they compare direct (Gauß) and iterative algorithm (Gauß-Seidel).
- get to know some constants and use algebraic expressions to generate new functions from the given elementary functions (exponential, logarithm, sine, cosine etc)!
  Students define, build and evaluate new functions, e.g. sine and cosine taking arguments in degrees. They will need this skill later on.
- evaluate trigonometric and hyperbolic functions together with their inverse functions!
  Students study periodicity, observe approximations like $\sin x \approx x$, $\cos x \approx 1$ for $|x| \ll 1$ or investigate ways to compute inverse functions like $\text{arsinh } x = \ln(x+\text{sqrt}(x\text{^}2+1))$.
- find zeroes of explicit functions per nested intervals, regula falsi or per Newton-Raphson!
  Students compare cost/benefit ratios, i.e. number of evaluation of the given function vs speed of convergence of the different algorithms for different initial values or start points.
- integrate per trapezoidal and Simpson-rule!
  Students again compare the cost benefit ratio of the two algorithms, they check convergence for different functions to be integrated, they optimise the implementation.
- solve differential equations numerically!
  Students observe precision of the exact (if available) vs the approximated solution in dependence on the increment and again compare the cost benefit ratio of the algorithms of Euler, Heun, Euler-Cauchy and Runge-Kutta.
- solve systems of differential equations numerically!
  Students realise the vector versions of the above algorithms and solve e.g. the system describing predator and prey populations.

Further applications are planned, to make available for example topics in probability calculus, optimisation etc.
Further evaluation of the concept will show whether this is worthwhile.

3.4   Evaluation of numerics.pdf

Students of the second semester were asked to work with numerics.pdf. They did so for **one hour** on average. On an ad hoc questionnaire [17] they marked and assessed numerics.pdf on a scale from 5 (worst) to 1 (best) as follows:

property	mark	students remarks
lucidity	1,7	*pdf very good, pages too cramped, vector calculus good*
usability	2,1	*help too terse, partly inconsistent usage*
detailedness	3,0	*too terse, more documentation, use tool tips*

Students assessed their own learning success in each topics as follows:

topic	students ratings
precision	*demonstrative, convincing*
vector calculus	*pretty observably, graphics were better*
systems of linear equations	*Gauß comprehensible, Gauß Seidel not clear*
zeroes	*very clear, very good because unfeasible by hand*
integration	*no rating (lack of time/interest?)*
differential equations	*no rating (lack of time/interest?)*

On the ground of even this ad hoc evaluation we think it justified to proceed with the development of interactive mathematical documents.

3.5 Costs and Benefits

The freeware typesetting system TeX – to be more precise pdfTeX – allows to generate on all platforms pdf-documents. This is advantageous for the author:

- simply manage and update the documents (using your favourite ASCII editor)
- compact sources (which can be browsed by plugins like techexplorer)
- specify all internal and external links in the source text (for navigation, references etc)
- specify all JavaScript code in the source text (for interaction, algorithms etc)
- simply generate documents from one source both to be read from screen and to be printed

This is advantageous for the students:

- free client software (Adobe's Acrobat Reader, version 4 or later)
- compact, therefore fast download of documents
- best quality on screen and on paper
- all possible navigation features
- for documents untypical potential of interaction

4 Conclusion

Interactive lecture notes have to prove to be beneficial in every day mathematical education. More serious evaluation is needed. First reactions of students and results of the ad hoc evaluation presented above are encouraging. Also the preliminary test – another example of an interactive document – for the preliminary mathematics course will have to prove to be useful.

JavaScript does not support graphics. This inherent limitation prevents to fall in love with the possibility to embed JavaScript code in pdf documents and to overestimate the potential of this technique.

In the Department of Electrical and Electronic Engineering and Computer Science of Hochschule Bremen we strive to integrate physics and informatics into the mathematical education and vice versa (at least per hyperlinks to the own material or the material of colleagues). On the occasion of this integration we will insert interactive elements into materials for physics (e.g. interactive numerical examples) and for algorithms (e.g. interactive numerical examples for complexity).

However, parallel to introduction, test and deployment of new techniques we should assure the quality of teaching, e.g. by didactic workshops (what we already did), by hospitation (what we will do). It is again a commonplace that new techniques – especially our attempts to establish a notebook university – need new didactical considerations.

References:

[1] Anderson, J.R.: Kognitive Psychologie; Spektrum Verlag 1996 – http://www.informatik.uni-freiburg.de/~noelle/studium/_Anderson.html
[2] Daii, F.: Lebendige virtuelle Welten; Springer 1996
[3] Embacher, F., Oberhuemer, P.: mathe online; TU Wien – http://www.univie.ac.at/future.media/mo/
[4] Fradkin, L.: Algebra Tutor – A Basic Algebra Tutoring System; South Bank University 2001
[5] Glasersfeld, E. von: Konstruktivismus statt Erkenntnistheorie; in Dörfler W., Mitterer J. (Hrsg.): Konstruktivismus statt Erkenntnistheorie; DRAVA Klagenfurt 1998, 11–39
[6] Gorny, P. et al: Medienunterstütztes Studium der Informatik (MuSIK); Universität Oldenburg – http://www-cg-hci.informatik.uni-oldenburg.de/~musik/ http://OLLI.Informatik.Uni-Oldenburg.de/
[7] Härdle, W., Simar, L.: Applied multivariate Statistical Analysis; HU Berlin – http://ise.wiwi.hu-berlin.de/~sigbert/mmedia/
[8] Hagen, H.: Integrating TeX, MataPost, JavaScript and PDF; Pragma – http://www.tug.org/applications/pdftex/calculat.pdf
[9] Hammer, S., Iakimtchouk, V., Wagner, E., Härle, F., Neundorf, V.: Fourier-Reihen; TU Ilmenau – http://www.weblearn.hs-bremen.de/weblearn/lernmaterial/Medieninformatik/Four/fourier.pdf
[10] Loviscach, J.: Mathematik für Medieninformatiker; Hochschule Bremen (*'hand written' lecture notes*) – http://www.weblearn.hs-bremen.de/home_loviscach/Public/0101ss/mai
[11] Loviscach, J.: Link-List of Mathematical Learning programs; Hochschule Bremen – http://www.weblearn.hs-bremen.de/home_loviscach/Public/medlehre/lernsoftware.html
[12] Mevenkamp, M.: Mathematik I – IV; Hochschule Bremen (*(fast) Fourier-Transform, systems of differential equations, control applications*) – http://www.weblearn.hs-bremen.de/weblearn/Profs/Mevenkamp/Mai1
[13] Ong, J., Ramachandran, S.: Intelligent Tutoring Systems: The What and the How; – http://www.learningcircuits.org/feb2000/ong.html
[14] Ottmann, T.: Authoring on the fly; Universität Freiburg – http://ad.informatik.uni-freiburg.de/mmgroup.aof
[15] Radhakrishnan, V.V.: pdfscreen – manual; University of Auckland, New Zealand – http://www.river-valley.com/download/manual.pdf
[16] Rahtz, S.: Hypertext marks in LaTeX: the hyperref package; CTAN – ftp://ftp.dante.de/tex-archive/macros/latex/contrib/supported/hyperref/doc/manual.pdf
[17] Risse, Th.: Mathematik für Informatiker (MAI); Hochschule Bremen (*interaktive lecture notes*) – http://www.weblearn.hs-bremen.de/risse/MAI/
[18] Risse, Th.: Kryptographie -- ein Exkurs; Hochschule Bremen 2000 (*on line & on demand lecture*) – http://www.weblearn.hs-bremen.de/weblearn/WorkSpaces/Teleteaching2000/tegrity_offen/test_vorlesung
[19] Risse, Th.: Teleteaching/Telelearning – What We Still Have to Learn; in M.Auer, U.Ressler (Eds): Interactive Computer Aided Learning – ICL2000; Kassel University Press 2000 – http://www.weblearn.hs-bremen.de/risse/papers/ICL2000
[20] Risse, Th.: Numerische Algorithmen / numerical algorithms; Hochschule Bremen 2001 – http://www.weblearn.hs-bremen.de/risse/MAI/docs/numerics.pdf
[21] Risse, Th.: Vorkurs-Test; Hochschule Bremen 2001 – http://www.weblearn.hs-bremen.de/risse/MAI/docs/vorkurs.pdf
[22] Story, D.P.: e-Calculus, Algebra in 10 Lessons, Web and Exerquiz Packages; – http://www.math.uakron.edu/~dpstory/
[23] Knuth, D.: TeX; Lamport, L.: LaTeX – http://www.dante.de/ http://www.tug.org/
[24] Thành, H.T., Rahtz, S., Hagen, H.: The pdfTeX manual; – http://tug.org/applications/pdftex/pdftex-a.pdf

[25] TeX and Acrobat -- better than powerpoint? –
http://www.utopiatype.com.au/products/demoseminar.pdf
[26] Urban-Lurain, M.: Intelligent Tutoring Systems – An Historic Review in the Context of the
Development of Artificial Intelligence and Educational Psychology; Michigan State University –
http://aral.cse.msu.edu/Publications/ITS/its.htm
[27] Weisstein, E., Wolfram, S.: MathWorld[1] – substitutional for *Mathematica course ware* –
http://mathworld.wolfram.com/
[28] Wolfram, S.: Mathematica; –
http://www.wolfram.com/, http://www.wolfram.com/products/webmathematica/
[29] Geonet – a dynamic geometric construction program; University of Bayreuth
http://www.did.mat.uni-bayreuth.de/geonet/

Author:

Thomas Risse, Prof. Dr.
Institute for Informatics and Automation, IIA, Hochschule Bremen
Neudtadtswall 30, D-28199 Bremen, Germany
risse@informatik.hs-bremen.de

---

[1] MathWorld web site removed by law from public view as of October 23, 2000. cp Legal
Documents http://mathworld.wolfram.com/docs/legal.html